Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students presenting for Further Mathematics Examination 2 in 2001 was 19 609, an increase on the 16 450 who sat in 2000. The selection of modules by students in 2001 is shown in the table below.

MODULE	%
1 Number patterns and applications	54
2 Geometry and trigonometry	84
3 Graphs and relations	50
4 Business-related mathematics	71
5 Networks and decision mathematics	36

The paper was generally regarded as quite accessible. Most students were able to complete the core material data analysis questions and were able to make a good start on each of their three modules. In 2001, there appeared to be a decrease in the number of students who answered fewer than three modules. Even so, about 50 students attempted only some of the core material questions without any attempt at the modules.

Within each module, questions centred upon a common theme and tended to become increasingly more challenging in the latter stages.

Core material questions required some analysis of given data. Students should be able to interpret statistics with which they are working as well as being able to obtain required results. Student answers in describing their observations and comparisons were quite varied in length and content but generally, they addressed the essential requirements of the questions.

Areas of weakness

Students need to read questions carefully and ensure that they attempt to answer all of any question as required. The requirement for rounding of answers was ignored by a number of students. Sometimes, answers were not reasonable and, upon checking this, should have been clear to the student.

Teachers might suggest to their students an examination technique that incorporates the following:

- Read the question carefully (some students underline important information).
- Ensure that working is clearly shown (if applicable) and follows a logical path (including labelled diagrams as appropriate).
- Check that the answer is reasonable, such as could the height of a kite really be longer than the attached string?
- Once finished, reread the question to ensure that all that aspects of the question had been answered.

Algebra and graphing skills of this year's cohort were reasonably sound with respect to course content. However, some students showed confusion about which formula to apply and when, especially in the Numbers, patterns and applications and Geometry and trigonometry modules.

There was evidence that some students bring 'quick fix' formulas in their pages of notes. The transposed form of the cosine rule could be quite useful in applications which require the angle to be found in the Geometry and trigonometry module. On the other hand, some tried to apply more exotic transpositions, that they did not appear to understand, as a possible 'quick fix' to a problem. This occurred particularly in the Business-related mathematics module.

Such transpositions may well be of assistance to a student: who understands the formula and its application, can substitute the correct values into the correct places and perform the necessary order of operations on the calculator, with use of brackets as appropriate. However, care needs to be taken to ensure that this is the case in practice.

For example, one student quoted $r = \left(\left(\frac{A}{P} \right)^{\left(\frac{1}{n} \times t \right)} - 1 \right) \times \left(n \times 100 \right)$ and then was unable to apply the formula.

Students continue to show a poor understanding of order of operations. Teachers might usefully alert students to the difference, for example, between $0.95^{26} - 1$ and $0.95^{(26-1)}$ as input for calculating the value of 0.95^{n-1} where $n = 26$.

Trial and error iteration is still a relatively popular and applicable method of finding a solution to a more complex numerical problem. An increasing use of graphic calculator functions is evident, whether by tabulation, graphical interpretation or through use of inbuilt functions. This was especially evident in the Numbers, patterns and applications and Business-related mathematics modules.

Some models of graphics calculators readily calculate solutions to time value money (TVM) function questions which might otherwise require substitution into the annuities formula. Students need to be familiar with the correct use of such functions when they choose to use them. While students found aspects of analysis associated with critical path contents in the Networks and decision mathematics module challenging, it was also noted that students did not readily access the early marks in Question 1. The concept of a *cut* and its application clearly require further attention.

Marking policy

For 2001, a single mark was deducted for the first instance of writing an answer that was not correct to the required number of decimal places. All subsequent, similar errors were not penalised.

Some students continue to write numerical answers only, without any working shown. While a correct answer will usually score the full marks for a question, this practice is often to the student's detriment. Without associated working or explanation, an incorrect or inaccurate answer will have no access to method or consequential marks that may be available for a particular question. Teachers must encourage student's to clearly show their working out, as appropriate. Assessors are expected to follow through a student's calculation and to award method marks as applicable where the student has shown an understanding of the solution process required for that question. Assessors are not permitted to make any assumptions about a student's working and only mark what is presented. For example, a method mark may apply to a question that asks for the interest earned on $2000 at 12% PA simple interest invested for two years. Consider the student who wrote an answer of $240 which is obviously only for one year and is wrong. A method mark could be awarded if the student had clearly shown that his/her answer was derived from $2000 \times 12\%$.

In 2001, consequential marks were available in some instances where answers to previous questions, necessary to continue with an answer to the current question, were wrong. It should be noted that these marks could only be allocated if the student clearly showed working out that used the previously incorrect answer.

SPECIFIC INFORMATION

Core – Data analysis

Question 1 (1.53/2)

a. (Average mark 1.89/Available marks 2)

ai.

4

aii.

socialising

Most students answered both of these correctly.

b. (1.53/2)

Socialising is mostly observed in the morning (53%) while travelling is mostly observed in the afternoon (61%) and feeding in the evening (71%)

Students were asked to test the assertion that dolphin activity is associated with time of day which is evidenced by finding clear examples of an association between two variables. 'Dolphin activity' is the dependent variable and 'Time of day' is the independent variable. As a general rule, an association (relationship) exists if 'Dolphin activity' varies in relation with changes in 'Time of day' and not just randomly.

Students could establish this by either:

for one activity, quoting and comparing at least two percentages or one percentage and shape of spread

OR

comparing the pattern of activity for two different times of day.

Several students read the table as referring to *numbers* of dolphins rather than *percentages*.

Question 2 (4.11/6)

a.

0	7 7
1	0 4 5 8
2	0 1 4 7
3	4 6 9
4	1

Most students were able to complete the stemplot.

b.

Mean = 2.2 hrs and standard deviation = 1.2 hrs

While most students found the correct mean, many wrote down the *population* standard deviation (1.1) rather than the required *sample* standard deviation for this sample set of data. *Population* standard deviation is not an examinable concept in Further Mathematics.

c.
The average (mean) and spread (standard deviation) of hours of non-dreaming sleep is greater (3 times more) than the average and spread for hours of dreaming sleep.

Many students legitimately found and compared the median for each distribution as a measure of the *centre* of the data, despite already having the mean calculated. A number of students interpreted *spread* to be the range of the data. While this was accepted, students should consider standard deviation, where appropriate, or the IQR as being more comprehensive statistics when comparing distributions. Full consequential marks were available here for the correct interpretation of a comparison which included incorrect answers from Question 2b.

Question 3 (2.28/5)

a.
linear
The equation for the regression line was quoted here by many students while others suggested that 'weak' or 'positive' was the key *assumption*. The equation itself, or a description of the strength of the relationship, is *not* the key *assumption* about the relationship as required in this question.

bi.
hours of dreaming sleep

Generally, most students obtained this answer. 'Dreaming sleep' was also accepted as an answer here, although technically incorrect, as it is not a variable.

bii.
22.9

The question was generally not well answered. Many students simply converted 0.4784 to a percentage and did not understand that the question was asking for the coefficient of determination, r^2.

biii.
European hedgehog

While many students identified the correct point on the graph, this question caused some difficulty for others in relating the graph to the table. Consequently, many students named the incorrect animal. The ability to link graphical and tabular information is an important skill.

biv.
2 hours (2.1 was acceptable)

Most were able to correctly read off the required value residual in this question.

Module 1 – Number patterns and applications

Question 1 (3.64/5)

a.
0.45 km

b.
14.35 km

c.
16.6 km

d.
33 weeks

The four parts of Question 1 were generally well done by those who attempted them.

Question 2 (2.14/5)

a.
13:3 (65:15 was also acceptable)

Many students did not read this question correctly as an answer of 80:65 was all too common.

bi.
$10.12

Most students obtained this answer.

bii.
$1 100 550

Many students incorrectly used 27.5km as the length of road to be completed rather than the 14.5km which had to be sealed. Others misused the cost of $75.90 per metre as a cost per kilometre to get $1100.55 as the total cost if the correct length had been used.

Question 3 (1.42/5)

a.

Bonti – Transport costs decrease in value as *n* increases and the section being sealed gets closer to Bonti

A common response ignored the given formula for transportation costs and explored the ease with which one could drive on road sections that had already been fully sealed. This was then inappropriately used to justify Amlin as the base town.

b.

$C_n = 0.95 \times C_{n-1}$ where $C_1 = 2375$

Many students did not understand the use of a difference equation as opposed to the formula for the *n* th term of a sequence. Common, incorrect answers involved the use of 0.95^{n-1} and/or a product involving 2375.

c.

$659

Generally completed satisfactorily although rounding off to the nearest dollar was not done by many.

d.

$20 482

Generally not well done. Many students did not understand this application of the sum to a geometric sequence. Some used the arithmetic series formula while others tried variations of the formulas for geometric sequences or series involving *n*-1.

Module 2 – Geometry and trigonometry

Question 1 (2.76/4)

a.

66°

Generally well done, although a number of students confused trigonometric ratios.

b.

19 m

Some students did not identify a different triangle to that in Question 1a.

The application of Pythagoras' theorem to a triangle with a given hypotenuse continued to pose problems for a small percentage of students.

Question 2 (1.56/3)

a.

40°

Generally done satisfactorily, although a number of students found the complementary angle 50° instead.

b.

35.35°

Rounding off required answers correct to two decimal places. Several students gave answers in degrees and minutes. As this was an early question in the module, the answer of 35° 21' was accepted for full marks as it was deemed 'correct to two decimal places'. Full, consequential marks were available here for the correct application of an incorrect answer in Question 2a.

Question 3a–b (1.87/4) c–d (0.6/4)

a.

108 m

Many students incorrectly bisected the 125° angle and allocated 62.5° to each angle either side of BD before using trigonometric ratios. A number of others assumed that triangle ABE was a right angled triangle with two base angles of 45° and applied Pythagoras' theorem.

b.

Area = $2 \times \frac{1}{2} \times 40 \times 80 \times \sin 125$

Generally done satisfactorily. Very few missed the doubling of the area of one half of the kite. Many students continued with the incorrect angle assumptions explained above and used $A = \frac{1}{2}bh$ several times. Heron's formula, while not in the required content, could be used. However, the additional calculations provided more opportunities for error than use of Area = $\frac{1}{2}\ bc \sin A$.

c.

53°

Generally, this question was not well done. Many tried to apply trigonometric ratios with incorrect angles as indicated above. Method marks were available for some reasonable attempt at use of the sine rule.

d.

5897 cm^2

Most students did not correctly attempt, or complete, this question. A common error was to simply multiply $\frac{3}{2} \times 2621$. A few students went to an inordinate amount of trouble (for a 1 mark question) in first trying to determine lengths of the lines in the two kites before using multiple applications of $A = \frac{1}{2}bh$ for the area to the second kite.

Module 3 – Graphs and relations

Question 1 (3.09/6)

a.

$4600

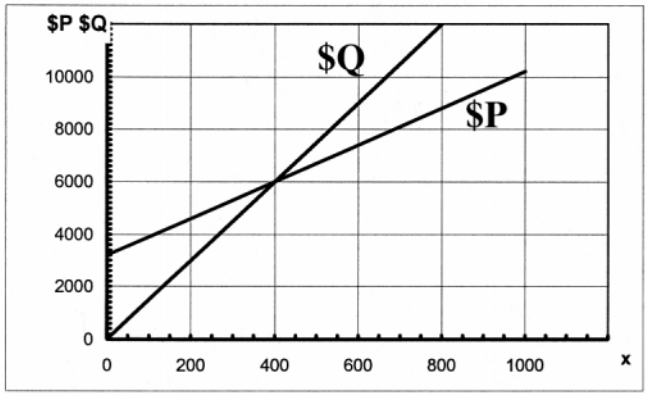
Generally done satisfactorily.

b.

$3000

The term *revenue* appears to be misunderstood by some students, as many gave $1600 for their answer. This was either the loss, after costs, or the profit if one ignores the fixed costs.

c.



Many students chose the points at $x = 0$ and at $x = 200$ to plot their lines. The relative closeness of these values magnified inaccuracies at values of $x > 200$ and led to incorrect intersection points for the two lines.

d.

x = 400

Full consequential marks were available for answers which were explicably derived from the intersection of the reasonable, but incorrect, two lines drawn for Question 1c.

Question 2 (4.41/9)

a.

Constraint 1:C
Constraint 2:B
Constraint 3:A

Most students got these answers although some simply copied the formulas listed next to the constraint numbers and given as part of the explanation to the question on the previous page.

b.

(570, 100) and R = 570, W = 100 was acceptable

Since reading off the graph was valid (for 1 mark each), an approximation of (570, 110) was also acceptable.

Some students wrote only one of the ordinates and consequently lost 1 mark. Several substituted $R = 570$ into the inappropriate formula $R + W \leq 700$ to get $W = 130$

c.

P = 8W + 10R

Generally completed satisfactorily.

d.
P_{max} = $6600
Some consequential marks were available here if working showed a correct substitution of at least two relevant points into their profit equation, written as an answer for Question 2c. A number of students selected points which were outside the feasible region, e.g. (570, 130).

Module 4 – Business-related mathematics

Question 1 (3.12/5)

ai.
$30 000

aii.
$16 875

b.
$24 000

c.
3 years

d.
20%
Parts a, b and d were generally answered satisfactorily. Part c was poorly done with a common incorrect answer of 2 years.

Question 2 (2.04/4)

a.
$180 000
Generally answered satisfactorily.

b.
$110 000
Generally answered satisfactorily. Consequential marks were available for the correct use of an incorrect answer to Question 2a.

c.
15.7%
Most students who successfully completed Questions 2a and 2b got this. Consequential marks were available for correct use of an incorrect answer to Questions 2a and/or 2b.

Question 3 (2.22/6)

a.
$76 800
Generally answered satisfactorily.

bi.
Q = 10 000, R = 1.085
The variables in the annuities formula continued to present difficulties for many students, particularly the value of *R*. Common, incorrect values were *Q* = 80 000 and/or *R* = 8.5.

bii
$32 527.68
The correct application of an appropriate finance function on the calculator would have found this answer without a written calculation being essential. However, this question was worth 2 marks and method marks are only available for a wrong answer if a calculation is shown. This was only awarded if a listing of the variables and relevant data used in the calculator finance function had been shown (with *n* = 10). If the student used the annuities formula, a method mark was available if the substitution of *n* = 10 and the answers to Question 1bi had been shown.

c.
$6800
Many students who attempted this question did not appear to connect the value of the scholarship with the value of the annual interest. A common incorrect answer was $10 000.
Module 5 – Networks and decision mathematics

Question 1 (0.83/3)

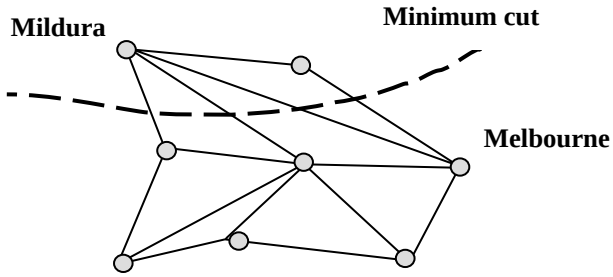
a.
This cut would indicate a maximum of 26 passengers could be on this route

The significance of a ‘cut’ in networks is not clearly understood by many students, despite it being a stipulated technical term. Some equated ‘cut’ with ‘cancelled’ in the context of the question and suggested that no passengers could therefore reach Melbourne. A simple numerical answer of **26** was accepted.

b.

15 passengers

Very few students found the minimum cut (as shown in the diagram) which gives the maximum flow in this question. Many apparently separated Mildura from every other town to find a ‘cut’ of 16. A method mark was only awarded for an incorrect answer if their ‘cut’ had also been clearly shown on the diagram. This question was not well answered in general with a mean score of less than 1 in 3 marks.



Question 2 a (0.73/1) b–d (2.19/5)

a.

Most students drew this line although some forgot the required arrow.

b.

Latest start time = 16, Earliest start time = 20

Very few students had both of these figures correct.

c.

A-B-C-F-H-J or A-B-C-F-G-I-J (both required)

While most students earned a mark here, very few students listed both of these paths

d.

25 weeks

Most students who got at least one of the answers to Question 2c also got this answer.

Question 3 (1.18/6)

a.

20 weeks

A consequential mark was available here for a number that was five fewer than their answer to Question 2d.

b.

23	**8000**	**F**	**10 000**	**2000**
22	**14 000**	**F, C**	**15 000**	**1000**

20	**29 000**	**C, E, F**	**25 000**	**-4000**

The completion of this table caused students difficulty. The following marking scheme was applied to this question.

A mark was allocated for each one of the following three points:

for showing a further reduction in F as well as a reduction in C and E *anywhere* in the middle column
for correctly completing the entire sequence in the Total Bonus column
for -4000 in the final cell of the Total Savings column.

c.

23 weeks as this gives a reduced time with maximum savings

Note 22 weeks gives the shortest time that still gives a benefit was an acceptable alternative interpretation but this had to refer to a correct and complete table. Only a few students got the expected answer, as for many their table was incorrect or incomplete. One consequential mark was available for an appropriate and correct comment in reference to an incorrect, but complete, table from Question 3b.